

July 23, 2013
hep-ph/0101139

Non-Annihilation Processes, Fermion-Loop and QED Radiation

GIAMPIERO PASSARINO*

*Dipartimento di Fisica Teorica, Università di Torino, Italy
INFN, Sezione di Torino, Italy*

The bulk of large radiative corrections to any process can be obtained by promoting coupling constants to be running ones and by including QED radiation at the leading logarithmic level via structure functions evolved at some scale. The problem of *fixing* the proper scale in running coupling constants and in structure functions for non-annihilation processes is briefly addressed and the general solution is analyzed.

Presented at the

5th International Symposium on Radiative Corrections
(RADCOR-2000)

Carmel CA, USA, 11–15 September, 2000

*Work supported by the European Union under contract HPRN-CT-2000-00149.

1 Introduction

At the eve of LEP shutdown it is of some importance to summarize the present status of high precision physics [1]. For $e^+e^- \rightarrow \bar{f}f$ all one-loop terms are known, including re-summation of leading terms. At the two-loop level leading and next-to-leading terms have been computed and included in codes like TOPAZ0 [2,3] and ZFITTER [4]. For realistic observables initial state QED radiation is included via the structure function method, or equivalent ones. Final state QED is also available as well as the interference between initial and final states [5]. Fine points in QED for $2 \rightarrow 2$ are as follows. For s -channel all the $\mathcal{O}(\alpha^2 L^n)$, $n = 0, 1, 2$ terms are known from explicit calculations, the leading $\mathcal{O}(\alpha^3 L^3)$ is also available and they are important for the studies of the Z lineshape.

Differences and uncertainties amount to at most ± 0.1 MeV on M_Z and Γ_Z and $\pm 0.01\%$ on σ_h^0 (MIZA, TOPAZ0 and ZFITTER) [6]. For non-annihilation processes (Bhabha) both structure-function and parton-shower methods have been analyzed and the uncertainty is estimated to be 0.061% from BHLUMI [7]. Certainly, full two-loop electroweak corrections are needed for GigaZ ($10^9 Z$ events) with a quest for a fast numerical evaluation of the relevant diagrams.

For $e^+e^- \rightarrow 4$ fermions all tree-level processes are available and $\mathcal{O}(\alpha)$ electroweak corrections are known only for the WW -signal and in double-pole approximation (DPA) [8] and [9]. $e^+e^- \rightarrow 4f + \gamma$ in Born approximation is also available for all processes [10].

Fine points in QED for $2 \rightarrow 4$ are as follows. For $e^+e^- \rightarrow WW \rightarrow 4f$ DPA gives the answer but, for a generic process $e^+e^- \rightarrow 4f$ QED radiation is included by using s -channel structure-functions, i.e. in leading-log approximation. The latter are strictly applicable only if ISR can be separated unambiguously. Otherwise their implementation may lead to an excess of radiation. Preliminary investigations towards non- s SF by GRACE and by SWAP [10] gives an indication on how to implement the bulk of the non-annihilation effect but still represent *ad hoc* solutions. These methods, which are essentially based on a matching with the soft photon emission, still contain an ambiguity on the energy scale selection with consequences on the predicted observables.

2 Non-Annihilation processes

There are several processes, namely those with t -channel photons that are not dominated by annihilation. Typical examples are single- W production and two-photon processes. The main question can be summarized as follows: how to include the bulk of radiative corrections?

At the Born level we still require the notion of input parameter set (IPS, i.e. the

choice of some set of input parameters (improperly called renormalization scheme (RS) in the literature) and of certain relations among them, e.g.

$$s_\theta^2 = 1 - \frac{M_W^2}{M_Z^2}, \quad \alpha \equiv \alpha_{G_F} = 4\sqrt{2} \frac{G_F M_W^2 s_\theta^2}{4\pi}, \quad (1)$$

Roughly speaking the theoretical uncertainty associated with the choice of the RS is most severe whenever low- q^2 photons dominate.

The first step in getting the right scales is represented by the Complex-Mass Renormalization in the Fermion-Loop approximation which gives [11]

$$\text{Couplings} \quad \Longrightarrow \quad \text{Running Couplings}$$

$$\text{Transitions} \quad \Longrightarrow \quad \text{Diagonal Propagator-Functions}$$

showing a pole in the 2nd sheet, and

$$\text{Born Vertices} \quad \Longrightarrow \quad \text{one fermion-loop corrected Vertices}$$

A typical example is shown by the following identities among diagrams:

The diagram shows two identities. The first identity is for a photon vertex: a vertex with two incoming fermion lines and a photon line (gamma) is equal to the sum of a vertex with a photon loop (gamma-gamma) and a vertex with a Z loop (Z-gamma). The second identity is for a Z boson vertex: a vertex with two incoming fermion lines and a Z boson line (Z) is equal to the sum of a vertex with a photon loop (gamma-Z) and a vertex with a Z loop (Z-Z). Open circles represent re-summed propagators, and dots represent vertices.

Here open circles denote re-summed propagators and the dot a vertex.

Running of coupling constants is shown in Fig. 2. In Fig. 2 the running of $e^2(q^2)$ is shown for $q^2 \rightarrow 0_+$, compared with the fixed value in the G_F -scheme. Furthermore, the evolution of $g^2(q^2)$ is shown for q^2 time-like or space-like.

The sizeable difference that one gets between e^2 running in t channel and e^2 fixed in the G_F -scheme is one of the major improvements induced by the FL-scheme in non-annihilation, Born processes.

However, the original formulation of the FL-scheme works only for conserved external currents. The extension to external massive fermions exists [13] and requires one additional replacement: one perform the calculation in the $\xi = 1$ gauge, neglects contributions from unphysical scalars and uses

$$\delta_{\mu\nu} \text{ (in propagators)} \quad \Longrightarrow \quad \delta_{\mu\nu} + \frac{p_\mu p_\nu}{M^2(p^2)}, \quad (2)$$

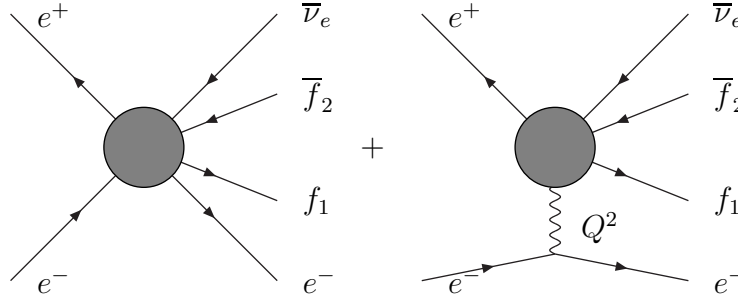
where $M(p^2)$ is the (complex) running mass. The connection with complex-poles, p_W, p_Z (here only for a massless internal world) is simple

$$\begin{aligned}
W &\implies M^2(p^2) = \frac{g^2(p^2)}{g^2(p_W)} p_W, \\
Z &\implies \frac{1}{M_0^2(p^2)} = \frac{g^2(p_Z)}{g^2(p^2)} \frac{c^2(p^2)}{c^2(p_Z)} \frac{1}{p_Z} \\
&\implies \frac{1}{M_0^2(p^2)} = \frac{c^2(p^2)}{M^2(p^2)}
\end{aligned} \tag{3}$$

and gives the M(assive)FL-scheme, where gauge invariance is respected and collinear regions, e.g. outgoing electrons at zero scattering angle, are accessible for safe theoretical predictions.

3 Applications to single- W

The single- W production mechanism is represented in the following figure.



The CC20 family of diagrams with the explicit component containing a t -channel photon.

The main consequences of applying the MFL-scheme are as follows:

- there is a maximal decrease of about 7% in the result if we compare with the G_F -scheme predictions but,
- the effect is rather sensitive to the relative weight of multi-peripheral contributions and is process and cut dependent [12].

4 QED radiation for arbitrary processes

Here the relevant question can be formulated as follows: is multi-photon radiation a one-scale or a multi-scale convolution phenomenon?

$$\sigma(p_+ p_- \rightarrow q_1 \dots q_n + \text{QED}) \stackrel{?}{=} \int dx_+ dx_- D(x_+, ?) D(x_-, ?) \times \sigma(x_+ p_+ x_- p_- \rightarrow q_1 \dots q_n) \quad (4)$$

In the above equation the question mark means that the corresponding scale has to be guessed. We need to understand how the standard SF-method is related to the exact YFS exponentiation. In the standard YFS treatment of multiple photon emission we have

$$\sigma\left(p_+ + p_- \rightarrow \sum_{i=1,2l} q_i + \sum_{j=1,n} k_j\right) \sim \int dPS_q |M_0|^2 E\left(p_+ + p_- - \sum_i q_i\right), \quad (5)$$

where E is the spectral function defined by

$$\begin{aligned} E(K) &= \frac{1}{(2\pi)^4} \int d^4x \exp(iK \cdot x) E(x), \\ E(x) &= \exp\left\{\frac{\alpha}{2\pi^2} \int d^4k e^{ik \cdot x} \delta^+(k^2) |j^\mu(k)|^2\right\} \end{aligned} \quad (6)$$

At this point we choose an alternative procedure where we do not separate the soft component from the hard one and compute some exact result valid for an arbitrary number of dimensions n and for on-shell photons, i.e. $k^2 = 0$,

$$I = \int d^n k e^{ik \cdot x} \frac{\delta^+(k^2)}{p_i \cdot k p_j \cdot k} \quad (7)$$

In dimensional-regularization one has the following result, valid $\forall x^2$:

$$I(x) = -\pi \rho \int_0^1 \frac{du}{P^2} \left(\frac{1}{\tilde{\varepsilon}} + 2 \ln 2 - \ln x^2 - \xi \ln \frac{\xi + 1}{\xi - 1} \right), \quad (8)$$

where we have defined a variable ξ as the ratio

$$\xi = \frac{|x_0|}{r}, \quad (9)$$

with an infinitesimal imaginary part attributed to x_0 ,

$$x_0 \rightarrow x_0 + i\delta. \quad \delta \rightarrow 0_+. \quad (10)$$

Furthermore, P is the linear combination

$$P = p_j + (\rho p_i - p_j) u, \quad (11)$$

where we have defined ρ to satisfy

$$(\rho p_i - p_j)^2 = 0, \quad (12)$$

and x_0, r are rewritten in covariant form as follows:

$$x_0 = -\frac{P \cdot x}{\sqrt{-P^2}}, \quad r^2 = x_0^2 + x^2. \quad (13)$$

The last integral shows the infrared pole $\frac{1}{\hat{\varepsilon}}$ and a collection of Li_2 -functions. Therefore, $E(K)$ is not available in close form. The scheme that we want to propose defines a coplanar approximation [14] to the exact spectral function,

$$\begin{aligned} I_{ij}^c &\stackrel{\text{def}}{=} -\frac{2}{3} \pi \rho_{ij} \mathcal{F}_{\text{cp}} \frac{1}{p_j^2 - \rho_{ij}^2 p_i^2} \ln \frac{\rho_{ij}^2 p_i^2}{p_j^2}, \\ I_{ii}^c &\stackrel{\text{def}}{=} -\frac{2}{3} \pi \rho_{ij} \mathcal{F}_{\text{cp}} \frac{1}{m_i^2}, \\ \mathcal{F}_{\text{cp}} &= \ln \left\{ e^{-\Delta_{\text{IR}}} \frac{p_i \cdot x p_j \cdot x}{m_i m_j} \right\}, \\ \Delta_{\text{IR}} &= \frac{1}{\hat{\varepsilon}} + \text{constants}. \end{aligned} \quad (14)$$

Within the coplanar approximation we have

$$\begin{aligned} E^{\text{pair} <ij>}(K) &\stackrel{\text{cp}}{\rightarrow} \frac{1}{(2\pi)^2} \left\{ \frac{e^{-\Delta_{\text{IR}}}}{m_i m_j} \right\}^{-\alpha A_{ij}} \frac{1}{\Gamma^2(\alpha A_{ij})} \\ &\times \int_0^\infty d\sigma d\sigma' (\sigma\sigma')^{\alpha A_{ij}-1} \delta^4(\sigma p_i + \sigma' p_j - K). \end{aligned} \quad (15)$$

This results explains why we have introduced the term *coplanar*. Note that $\alpha A \sim \beta$ only when the corresponding invariant is much larger than mass² but the above expression is valid for all regimes and it is easily generalized to n emitters with the result that ¹ in a process $2 \rightarrow n$ any external charged leg i talks to all other charged legs, each time with a known scale s_{ij} and with a known total weight proportional to

$$x_i^{\alpha(A_1^i + \dots + A_I^i) - 1} / \Gamma(\alpha(A_1^i + \dots + A_I^i)), \quad 0 \leq x_i \leq 1 \quad (16)$$

¹A.Ballestrero, G.P. work in progress

Note that each A has the appropriate sign, in/out, part/antp. Furthermore, $I(i)$ is the number of pairs $\langle ij \rangle$ with i fixed. The IR exponent is given by

$$\alpha A = \frac{2\alpha}{\pi} \left\{ \frac{1+r^2}{1-r^2} \ln \frac{1}{r} - 1 \right\}, \quad \frac{m_e^2}{|t|} = \frac{r}{(1-r)^2} \quad (17)$$

For Bhabha scattering we will have the following combination:

$$-A(s, m_e) - A(t, m_e) + A(u, m_e) = \frac{2}{\pi} \left[\ln \frac{st}{m_e^2 u} - 1 \right], \quad (18)$$

obtained as an exact result, not a guess.

5 Conclusions for QED

The structure-function language is still applicable but initial state structure functions evaluated for one scale is, quite obviously, not enough. In any process each external leg brings one structure function; since all charged legs talk to each other, each SF is not function of one *ad hoc* scale but all $\langle ij \rangle$ scales enter into SF_i . The exact spectral-function is a convolution of SF

$$\begin{aligned} E^{\text{pair} \langle ij \rangle}(K) &= \int d^4 K' \Phi(K') E_{\text{cp}}^{\text{pair} \langle ij \rangle}(K - K'), \\ \Phi(K) &= \frac{1}{(2\pi)^4} \int d^4 x \exp \{i K \cdot x + \alpha (I - I_{\text{cp}})\} \\ &= \delta(K) + \mathcal{O}(\alpha) \end{aligned} \quad (19)$$

Furthermore, IR-finite reminders and virtual parts can be added according to the standard approach of reorganizing the perturbative expansion.

Acknowledgments

I am grateful to A. Ballestrero for the collaboration in this research and to H. Haber for the invitation and for the very pleasant stay at Carmel.

References

- [1] D. Bardin, M. Grunewald and G. Passarino, hep-ph/9902452.
- [2] G. Montagna, F. Piccinini, O. Nicrosini, G. Passarino and R. Pittau, Comput. Phys. Commun. **76** (1993) 328.

- [3] G. Montagna, F. Piccinini, O. Nicrosini, G. Passarino and R. Pittau, Nucl. Phys. **B401** (1993) 3.
- [4] D. Bardin, P. Christova, M. Jack, L. Kalinovskaya, A. Olchevski, S. Riemann and T. Riemann, hep-ph/9908433.
- [5] S. Jadach, B. F. Ward and Z. Was, Comput. Phys. Commun. **130** (2000) 260 [hep-ph/9912214].
- [6] LEP EWWG/LS 2000-01.
- [7] S. Jadach, W. Placzek, E. Richter-Was, B. F. Ward and Z. Was, Comput. Phys. Commun. **102** (1997) 229.
- [8] A. Denner, S. Dittmaier, M. Roth and D. Wackeroth, Nucl. Phys. **B587** (2000) 67 [hep-ph/0006307].
- [9] S. Jadach, W. Placzek, M. Skrzypek, B. F. Ward and Z. Was, hep-ph/0007012.
- [10] M. W. Grunewald *et al.*, hep-ph/0005309.
- [11] W. Beenakker *et al.*, Nucl. Phys. **B500** (1997) 255 [hep-ph/9612260].
- [12] G. Passarino, Nucl. Phys. **B578** (2000) 3 [hep-ph/0001212].
- [13] G. Passarino, Nucl. Phys. **B574** (2000) 451 [hep-ph/9911482].
- [14] C. Chahine, Phys. Rev. **D18** (1978) 4617.

